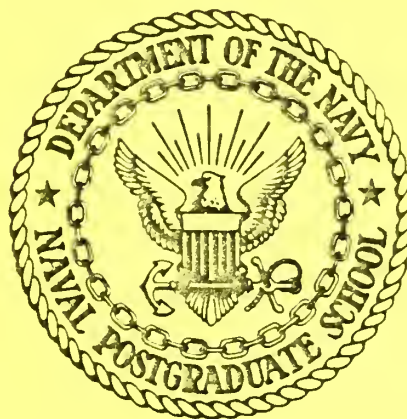


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POSSIBLE APPROACHES TO DETERMINING LATERAL  
AND RANGE EFFECTS OF BOMB STATIONS,  
BASED ON OBSERVED IMPACT POINTS

by

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POSSIBLE APPROACHES TO DETERMINING LATERAL AND RANGE EFFECTS  
OF BOMB STATIONS, BASED ON OBSERVED IMPACT POINTS

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ABSTRACT

Two simple experimentation approaches to determining the effects of bomb rack positions on bomb impact offsets and relative range errors are described. The approaches use only impact data obtained through the prescribed experimental procedures. They do not require delivery aircraft track data nor aircraft velocity and acceleration data. Statistical analyses required to test the significance of the rack positions as well as estimate the magnitudes of the effects are discussed.



## 1. INTRODUCTION

At the TPQ/27 PSVT planning meeting held in Monterey (19 Jan 78), a question arose concerning whether the rack position of a bomb affected its expected impact point relative to the target. At that time it was suggested that an experiment of very modest size could be performed which would provide an answer to this question. The purpose of this paper is to suggest two approaches through which such an experiment could be run. We refer to these as the "ditch in the desert" approach and the "pit in the Pacific" approach.

The main feature of the proposed approaches is their relatively low demand on experimentation resources. In one approach the aircraft need not be tracked; in the other a TPQ 10 delivery system (or a system with equivalent capability) could be used. In neither case would it be necessary to track bombs or to measure delivery aircraft velocities and accelerations for use in subsequent analysis of observed impact points. Bomb impact locations are, of course, required with both approaches.

The goal of the proposed experiment is to determine whether rack positions "cause" (that is, are associated with) significant effects in bomb impact offset and range. Significance is defined here in terms of the signal-to-noise ratio. If the effect of a given rack position is as much as 15 percent of the system CEP, this effect will be detected with fairly high probability if 10 sorties are flown (see references 1 and 2).



## 2. "Ditch in the Desert" Approach

The basic idea with this approach is to estimate possible offset and range effects of rack positions using stick bombing data. For each stick, the observed impact points would be used to estimate the ballistic trace of the aircraft path. Deviations left and right of this line would provide offset data; deviations along this line (described more fully below) would provide range error data. Such data for each symmetric pair of rack positions would be accumulated over the proposed 10 sticks, resulting in 20 observations of offset and range error outcomes. These data would be used to test the significance of the rack position, relative to system error characteristics. If the test rejects the null hypothesis of no effect, these data would be used in addition to estimate the magnitude of the effect, for the delivery conditions used in the experiment.

Assumptions of this approach include:

- a) The target area is a flat plane parallel to the aircraft paths.
- b) The bombs in a stick are released with a precise time spacing between them. This spacing should be large enough so that impacts on the ground occur in the same sequence as the release sequence. It is desirable to use a time spacing that:
  - i) gives a "long" stick (to facilitate estimation of the ballistic aircraft trace), and
  - ii) helps eliminate perturbations in aircraft flight due to release of previous bombs (perhaps the auto pilot would be useful).



- c) The aircraft path is a straight line during the release period.
- d) The ballistic trace of the aircraft path on the target plane is a straight line.
- e) Impact points can be determined (and recorded) without significant error.

The experiment would involve laying 10 sticks under as nearly the same aircraft speed and altitude drop conditions as practical. (Minor variations are OK; variations would merely make the apparent system noise level slightly higher, so the test would lose somewhat in its ability to detect rack position effects if the latter are present.) Of course, wind and air density profiles would not be the same for the 10 sticks, nor is it necessary to use the same aircraft heading for the sticks.

The impact points from a given stick might appear as shown in Figure 1, when plotted. The line shown in Figure 1 is the

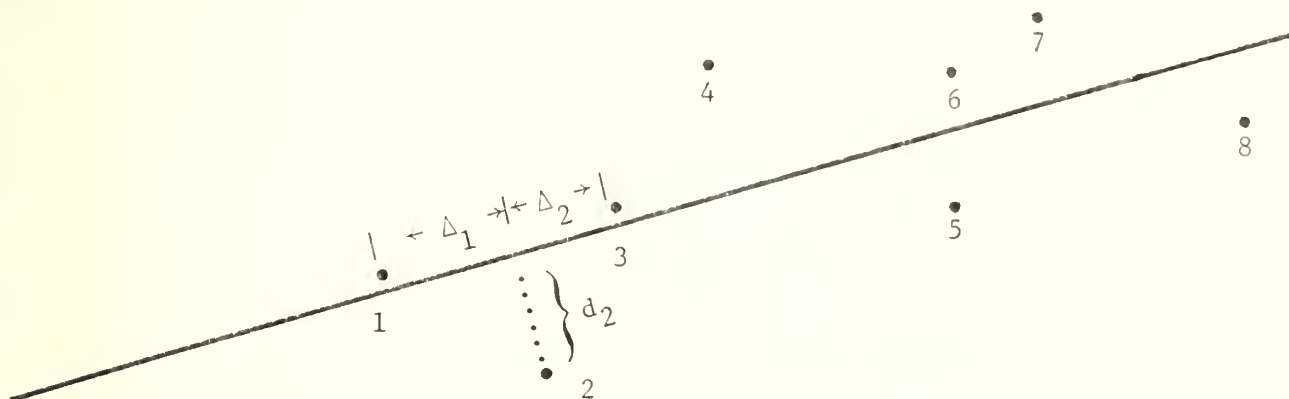


Figure 1. Hypothetical impact points obtained in one stick.

The line shown is the estimated ballistic trace of the aircraft track. Numbers shown with impact points are rack positions.

estimated ballistic trace of the aircraft path on the target plane. It is estimated by statistically fitting a line to the drop points (not using regression, however, since there is not an "independent variable" nor a "dependent variable" for these data pairs). Possibly a line which minimizes the sum of squares of the perpendicular distances to the points would be satisfactory. In order to avoid possible bias in this estimation procedure, the drop sequences should be designed so that the bombs come off the aircraft in a symmetric pattern (or nearly so). Fortunately, such a sequence is probably the best for operational reasons as well.

The (perpendicular) distances from the impact points to this line (such as " $d_2$ " in Figure 1) provide the offset data to be used in subsequent analysis. The range error data can be obtained as follows. First, estimate the mean range gap between bombs in a given stick by the average,  $\bar{\Delta}$ , of the observed gaps  $\Delta_1, \Delta_2, \dots, \Delta_7$  formed by projection of the impact points onto the ballistic trace line ( $\Delta_1$  and  $\Delta_2$  are shown in Figure 1). Second, find eight points on the line, equally\* spaced  $\bar{\Delta}$  units apart, which minimize the sum of squares of distances between these points and the projections of the impacts on the line. The former points are the "estimated impact ranges"; the latter are "observed impact ranges." The signed distances between the estimated impact ranges and the observed impact ranges form the range error data for subsequent analysis.

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\* Or with theoretical spacing associated with ballistic computation in the TPQ/27 (if any).

Within a stick, the negative of each left position offset datum, together with its symmetric right position counterpart, form a pair of offset data. These are combined with the corresponding data pairs from the remaining 9 sticks, to form a set of 20 numbers. The hypothesis of no significant effect due to these rack positions (that is, the left and right symmetric pair) can be tested with a nonparametric statistical test, such as the sign test. (See Reference 3.) If, based on examination of the data, normal distribution theory appears tenable, a parametric test, based on an F-test or, perhaps, a t-test might be used. Similar tests can be used to test hypotheses concerning range effects (reference 3).

For cases in which there appears to be a significant effect, magnitude of the effect can be estimated using the offset and range error data. For example, the average value of the 20 estimated offset data for a given rack position symmetric pair is an estimate of offset effect due to that position. Range error effects can be estimated similarly.

### 3. "Pit in the Pacific" Approach

This approach uses bombs dropped one at a time at a target, in contrast with stick bombing data described above. Since this experiment might be run against an ocean target at Point Mugu, and all bombs in each sortee are dropped at a target, we call it

"pit in the Pacific." As mentioned earlier, the TPQ/10 might be used to control the bombing system on these target runs. Within each sortee, the aircraft heading should be nearly the same at the times of all bomb releases, so that out of each sortee would come a group of (say, 8) impact points. These impacts would share a common wind profile and other system inputs and environmental conditions. Each subsequent sortee would have its particular parameter set and environmental conditions, and thus might place its group differently with respect to the target. We describe below how these groups of impacts could be analyzed to test significance of rack positions.

The assumptions for this approach are:

- a) Over the duration of a sortee, environmental conditions do not change significantly.
- b) The bombs are dropped one at a time, from a "circular" flight pattern, against a target.
- c) Bomb impact locations, relative to the target, can be measured and recorded without significant error.
- d) The aircraft heading and altitude are nearly the same for all drops in a sortee. Altitude is nearly the same for all sortees.
- e) There is not an attempt to "tweek" the bombing system, to improve its accuracy, within a sortee.
- f) The aircraft heading used for each drop set within a sortee is known, to within  $\pm 5^\circ$ .

The experiment is run by flying 10 sorteers against the target, and measuring impact position for each drop. If a system such as the TPQ/10 is used, for which measured drop conditions can be used to account for a portion of each predicted impact point, the impact data should be adjusted to take this delivery error into account. The data resulting from 10 sorteers should be 10 groups of eight impacts (adjusted, if possible as discussed above). Within each group, the rack position associated with each impact must be recorded. For each group, the aircraft heading should be known (to a reasonable accuracy).

To analyze such data, we would first transform the impact data to a coordinate system based on aircraft heading (positive y-axis) and cross range (right misses plotted toward the positive x-axes). For each group, the center of impact is estimated, and the coordinate system is translated so as to place this point at the origin. Within each group, for each pair of impacts associated with a symmetric pair of rack positions, the negative of the range error component associated with the left rack position, together with the range error component for the symmetric corresponding right rack position, form a pair of range "miss" data. Pooled over the 10 sorteers, we thus obtain a set of 20 range error estimates for that rack position. A hypothesis of no range error effects due to rack position can be tested with a nonparametric, or if tenable, a parametric test as described for the earlier approach. Similarly, we obtain sets of 20 data points for testing the significance of rack position on lateral "miss."



#### 4. Discussion and Comments

Using either of the approaches described above, we can obtain a test of whether rack position has a significant effect on bomb impact lateral offset or range error. The data analyses described are based on well known and widely accepted statistical procedures. Either procedure should, with reasonable confidence, detect differences (if any) on the order of 15% of CEP or more. (See Appendix I in which an earlier paper, concerning estimation of sample size requirements, is reproduced.) For practical purposes, differences smaller than .15 CEP should not adversely affect the TPQ/27 PSVT.

It is perhaps worth pointing out that failure to find statistically significant effects due to rack position would be in itself an important finding. Thus, the prediction by some that the proposed experiment "would not show anything" may not be correct, even in the event that no significant differences were detected.

Of the two approaches discussed above, the author slightly prefers the "Pit in the Pacific" approach, because

- a) it simulates the drop procedure (one at a time) to be used in the PSVT, and so gives data relevant to the PSVT assessment; and
- b) it avoids possible errors in the stick bombing approach, caused by aircraft flight perturbations due to earlier bomb releases.

This approach does, however, introduce more error into the drop data, due to bomb system delivery errors, than would the "ditch in the desert" approach. Subject to tenability of the assumptions listed for each approach, both approaches appear quite feasible from the statistical point of view.

Finally, we remark again that these approaches make no use of aircraft track information, nor of aircraft velocity and acceleration information, nor of bomb trajectory information. Instead, the approach we suggest "shoots out" (in the sense of artillery adjustment) effects that might be estimated by such elaborate data, by statistical analysis. This is done at the expense of requiring a somewhat higher sample size (about 40% higher) than would be required if proper use could be made of the aircraft tracking and motion data mentioned above. It appears to the author that the relatively high cost of obtaining aircraft tracking and motion data, and the cost of analyses required to utilize such data, make the general approach suggested in this report very attractive.



## REFERENCES

1. D. R. Barr, "Sample Size Curves for Determining Number of Runs Necessary in Rack Position/Impact Offset Study," Technical Note distributed Jan. 30, 1978; Reproduced Appendix A of this report.
2. T. T. Bean, Message R031620Z Mar 78 to RUWDPAA/MAD Pt Mugu, CA.
3. Dixon and Massey, "Introduction to Statistical Analysis," McGraw-Hill Book Company, New York, 1969.

## APPENDIX I

SAMPLE SIZE CURVES FOR DETERMINING NUMBER OF RUNS NECESSARY  
in RACK POSITION/IMPACT OFFSET STUDYD. R. Barr  
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We wish to detect whether rack position of a bomb is associated with significant lateral displacement of the impact. Using symmetry, each run gives  $s$  observations from each off center rack location. Suppose the deflection "due" to rack position  $j$  is  $d_j$ . We wish to test whether  $d_j = 0$ , against the alternate hypothesis  $d_j > 0$ . This is accomplished using a test statistic based on the observed displacements from the mean, overall runs:

$$\bar{D}_{\cdot j \cdot} = \frac{1}{2\ell} \sum_{i,k} (D_{ijk}) , \quad \text{where } D_{ijk} = X_{ijk} - \bar{X}_{\cdot \cdot k};$$

$X_{ijk}$  is  $i^{\text{th}}$  drop from position  $j$  in run  $k$ ;  
 $i = 1, 2, \dots, s$ ;  $j = 1, 2, 3$ ;  $k = 1, 2, \dots, \ell$ .

Under  $H_0$ ,  $E(\bar{D}_{\cdot j \cdot}) = 0$ ;  $V(\bar{D}_{\cdot j \cdot}) = \frac{\ell}{4\ell^2} [2 - \frac{1}{2s}] \sigma^2$ , where  $\sigma^2 = V(X_{ijk})$ ; and the  $X_{ijk}$ 's are assumed to be independent.

Under  $H_0$ ,

$$Z = \frac{\bar{D}_{\cdot j \cdot}}{\sqrt{[\frac{4s-1}{8\ell s}] \sigma^2}} \sim N(0,1) ,$$

where  $\sigma = CEP / \sqrt{2 \ln 2}$ .

Reject  $H_0$  (at  $100\alpha\%$  level) if  $Z > z_{1-\alpha}$ .

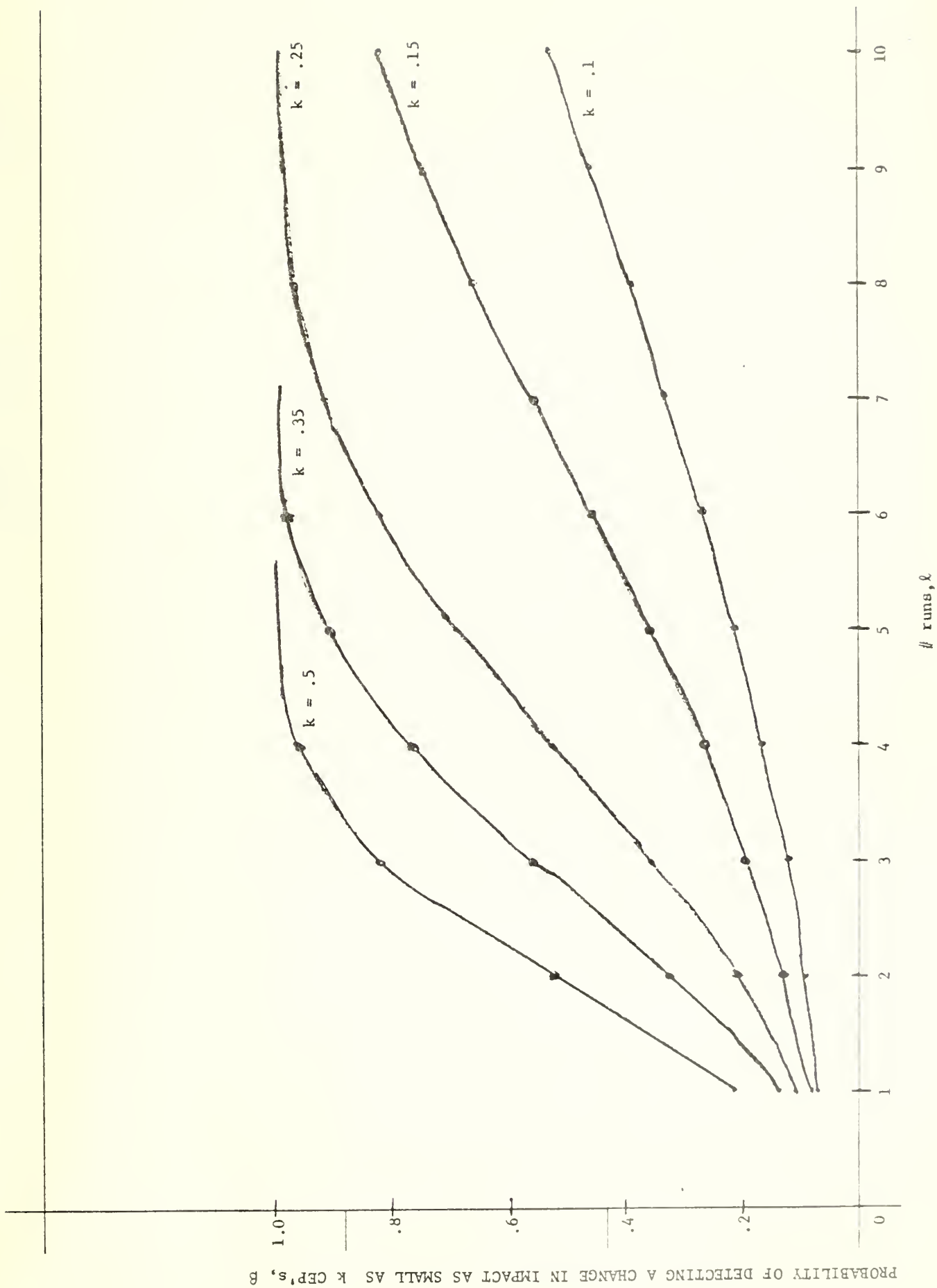
Now we detect a change of  $k \cdot \text{CEP}$  with probability  $\beta$  where  $P[Z > z_{1-\alpha} | \bar{ED}_{\cdot j} = k \cdot \text{CEP}]$ .  
 Let  $J = [\frac{4s+1}{8\ell s}] \sigma^2$ . Then

$$\begin{aligned} \beta &= P_1[Z > z_{1-\alpha}] = P\left[\frac{\bar{D}_{\cdot j}}{\sqrt{J}} - \frac{k \cdot \text{CEP}}{\sqrt{J}} > z_{1-\alpha} - \frac{k \cdot \text{CEP}}{\sqrt{J}}\right] \\ &= P\left[\frac{\bar{D}_{\cdot j} - k \cdot \text{CEP}}{\sqrt{J}} > z_{1-\alpha} - \frac{k \cdot \text{CEP}}{\sqrt{J}}\right] \\ &= P\left[Z' > z_{1-\alpha} - \frac{k \cdot \text{CEP}}{\sqrt{J}}\right], \quad \text{where } Z' \sim N(0,1), \\ &= 1 - \Phi\left(z_{1-\alpha} - \frac{k \cdot \sqrt{2 \ln 2}}{\sqrt{\frac{4s-1}{8\ell s}}}\right). \end{aligned}$$

A plot of the number of runs  $\ell$  versus the probability of detecting a difference as small as  $k \text{ CEP}'s$  is shown in the attached figure, for a test at significance level  $\alpha = .05$ , for several values of  $k$  ( $s$  is assumed to be 4).

Example: With  $\ell = 8$  runs, we will detect a change, due to rack position, as small as  $\frac{1}{4}$  CEP units with probability 0.96.

Note: Assuming  $s = 4$  is not critical. Any value between 2 and 7 will yield about the same curves.



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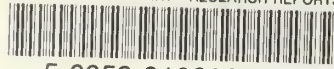
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